Math 241 Sample Problems for Final Exam

Question 1 Show that the following limit does not exist as $(x, y) \to (0, 0)$ by considering different paths of approach.

$$\lim_{(x,y)\to(0,0)} \frac{x^2y + y^2}{x^4 + y^2}$$

Question 2 Let $f(x, y) = \dfrac{\sin(2x - y)}{y}$. Find the equation of the tangent plane to the surface $f(x, y)$ at the point when $(x, y) = (2, 1)$.

Question 3 Let $z = g(x, y)$ and suppose that $x(t) = t^2 + 3t + 2$ and $y(t) = e^t + \sin(3t)$. Find $\left.\dfrac{dz}{dt}\right|_{t=0}$ if

$$\left.\frac{\partial g}{\partial x}\right|_{(1,2)} = 6,\ \left.\frac{\partial g}{\partial y}\right|_{(1,2)} = -2,\ \left.\frac{\partial g}{\partial x}\right|_{(2,1)} = -3,\ \left.\frac{\partial g}{\partial y}\right|_{(2,1)} = 8,\ \left.\frac{\partial g}{\partial x}\right|_{(0,0)} = 0,\ \left.\frac{\partial g}{\partial y}\right|_{(0,0)} = -4$$

Question 4 Let the temperature at a point (x, y) be given by $T(x, y) = \dfrac{xy}{(1 + x^2 + 2y^2)}$.
a) Find the direction in which the temperature rises most rapidly at $(1, 2)$.
b) Find the directional derivative of T at the point $(1, 2)$ in the direction of the vector $\mathbf{v} = 5\mathbf{i} - \mathbf{j}$.

Question 5 Let $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$.
a) Find the critical points of $f(x, y)$.
b) Classify the critical points in part a) as a relative maximum, relative minimum or saddle point.

Question 6 Lagrange multipliers?

Question 7 Find the volume of the solid wedge cut from the cylinder $4x^2 + y^2 = 16$ below by the plane $z = 0$ and above by the plane $z = y$ by evaluating an appropriate double integral.

Question 8 Evaluate the double integral $\displaystyle\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{(1 + x^2 + y^2)^{3/2}}\, dx\, dy$, by using polar coordinates.

Question 9 Express the triple integral: $\displaystyle\iiint_R \frac{1}{x^2 + y^2 + z^2}\, dy\, dz\, dx$ as an integral in spherical coordinates if R is the region bounded below by the paraboloid $2z = x^2 + y^2$, and above by the sphere $x^2 + y^2 + z^2 = 8$. This is a little tricky since you will need to use two triple integrals. Do NOT Evaluate the integrals!

Question 10 Let $\mathbf{F}(x, y) = (e^x \sin y - y)\mathbf{i} + (e^x \cos y - x - 2)\mathbf{j}$ be a vector field defined on $\mathbb{R}^2$.
a) Show that $\mathbf{F}$ is a conservative vector field.
b) Evaluate the line integral $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the path $\mathbf{r}(t) = (\ln(t + 1)\cos(\sqrt{\pi}\ t))\mathbf{i} + (t^2 + \frac{1}{2}\pi)\mathbf{j}$, $0 \le t \le \dfrac{\sqrt{\pi}}{2}$.

Question 11 Evaluate the line integral $\displaystyle\int_C (x + xy^2)\, dx + 2(x^2y - y^2 \sin y)\, dy$ where C is the path oriented counterclockwise enclosing the region in the first quadrant bounded by $y = x^2$ and $y = 1$ and $x = 0$ by using Green's Theorem.

Question 12 Use the transformation $x = u^{2/3}v^{1/3}$, $y = u^{1/3}v^{2/3}$ to find $\displaystyle\iint_R \frac{x^2 \sin xy}{y}\, dA$ where R is the quadrangular region bounded by the parabolas $x^2 = \frac{1}{2}\pi y$, $x^2 = \pi y$, $y^2 = \frac{1}{2}x$, $y^2 = x$. You may assume that $u, v > 0$.

Question 13 Compute $\displaystyle\int_C \mathbf{F} \cdot \mathbf{T}\, ds$ and $\displaystyle\int_C \mathbf{F} \cdot \mathbf{n}\, ds$ for the vector field $\mathbf{F}(x, y) = (x + y)\mathbf{i} - (x^2 + y^2)\mathbf{j}$ where C is the boundary of the triangle bounded by $y = 0$, $x = 1$ and $y = x$ oriented counterclockwise.